Automated adjustment of voxel-dependent importance factors in inverse planning

1 Introduction

For IMRT, the goal is to generate fluence maps for each beam such that the dose distribution satisfies certain constraints, usually dose-volume or maximum-dose. Traditionally, volumes of interest are weighted against each other by use of volume-wide importance factors. By assigning the PTV a higher importance than the organs at risk, the objective of giving the PTV its prescribed dose outweighs the desired zero dose in the OAR's. However, the solution heavily depends on the choice of these volume-wide importance factors. Generally, manual adaptation is a lengthy trial-and-error process.

A new approach is the use of voxel-dependent importance factors [1]. This provides more local control over the dose distribution. In this paper, an inverse planning algorithm is presented that is based on voxel-dependent importance factors that are automatically adapted. The constraints are prioritized to ensure correct PTV dosage and maximum sparing of important OAR's, at the expense of less important OAR's. With this approach it is also possible to obtain highly conformal dose distributions.

2 Materials and methods

To meet the imposed constraints a two-step algorithm is used. In the first step beam profiles are optimized according to a quadratic objective function, with the condition that the fluence is non-negative. In the second step, the result is evaluated and according to violated dose-volume or maximum-dose constraints, the voxel-coefficients for the objective function are adapted, and the first step is run again. This process is repeated until all the constraints are satisfied, or for a maximum number of iterations.

2.1 Objective function

The relation between the dose and the fluence (represented as a vector of beamlets) is linear and can be written as a matrix-vector product $\boldsymbol{d} = H\boldsymbol{f}$. The matrix H is the dose deposition matrix composed of the distribution vectors of all beamlets and is stored as a sparse matrix to reduce memory usage and increase speed when computing matrix-vector or matrix-matrix products. The quadratic objective function used in the beam profile optimization can be written as follows [2]:

$$s(\boldsymbol{f}) = \sum_{v \in \mathcal{V}} \xi_v (H\boldsymbol{f} - \boldsymbol{d}_v^p)^T \tilde{\eta}_v (H\boldsymbol{f} - \boldsymbol{d}_v^p) + \kappa (M\boldsymbol{f})^T (M\boldsymbol{f}), \quad \boldsymbol{f} \geq 0. \tag{1}$$

This function consists of two terms: a dose objective term and a smoothing term. The first term records for each voxel in a volume v the difference between the attainable dose $H\boldsymbol{f}$ and the prescribed dose $\boldsymbol{d}_v^p$, which is kept constant for all voxels in a certain volume, usually the prescribed dose for the PTV, and 0 for the organs at risk. The diagonal matrix $\tilde{\eta}_v$ contains the voxel-dependent importance factors. The different volumes of interest (representing structures) $v \in \mathcal{V}$ are weighted by volume-wide importance factors ξ_v.

The second term is the smoothing term, regulated by a smoothing factor κ. This term encourages the fluence to be smooth. The product $M\boldsymbol{f}$ is a discretization of the Laplacian (second derivative) of the fluence $\boldsymbol{f}$.

2.2 Beam profile optimization

Equation 1 can be written in canonical form:

$$s(\boldsymbol{f}) = \frac{1}{2}\boldsymbol{f}^T A \boldsymbol{f} + \boldsymbol{f}^T \boldsymbol{b} + c, \tag{2}$$

where:

$$A = H^T Q H + \kappa S, \qquad \boldsymbol{b} = H^T \boldsymbol{q}, \qquad c = \sum_v \xi_v (\boldsymbol{d}_v^p)^T \tilde{\eta}_v \boldsymbol{d}_v^p, \tag{3}$$

$$Q = 2\sum_v \xi_v \tilde{\eta}_v, \qquad S = 2M^T M, \qquad \boldsymbol{q} = -2\sum_v \xi_v \tilde{\eta}_v \boldsymbol{d}_v^p. \tag{4}$$

The advantage of this formulation is that the calculation of the score requires only a few matrix-vector computations and that there are many methods available for minimizing this function. The most time consuming computation is that of the first term of matrix A. We have developed an efficient sparse matrix algorithm to compute $Z = H^T Q$ by only transposing the rows of H if the corresponding element on the diagonal of Q is

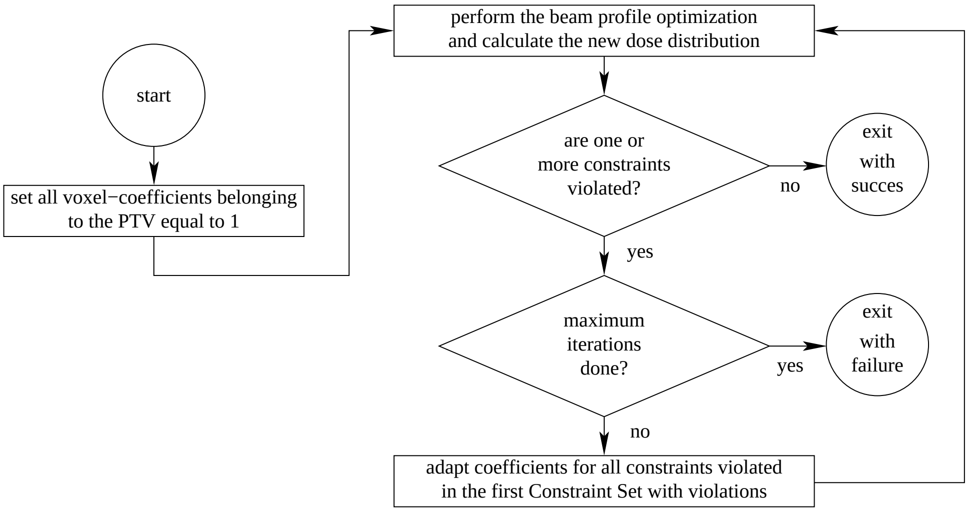


Figure 1: Flow diagram of the two-step voxel-coefficients optimization.

nonzero. The number of computations scales linearly with the number of non-zero elements on the diagonal of Q.

An IMRT problem using an objective function similar to equation 1 without the smoothing term is usually ill-posed because it is possible to obtain nearly identical dose distributions with different fluence maps [3]. Without precautions the resulting fluence will contain high frequencies due to the numerical noise in the problem. The numerical degeneracy of a matrix depends on the condition number, which is the ratio between the largest and smallest eigenvalue. By adding a well-conditioned matrix (such as the identity matrix or Laplacian) the smallest eigenvalue increases, so the condition decreases, which is favorable.

The Hessian (matrix A in equation 2) is positive definite, so minimizing $s(\boldsymbol{f})$ under the condition that $\boldsymbol{f} \geq 0$ can be solved by any convex quadratic minimization algorithm. Our algorithm of choice is BOXCQP [4] because this algorithm is easy to implement and faster than most quadratic programming algorithms. Furthermore it uses the Cholesky decomposition which is suitable for parallellization on SMP (symmetric multiprocessors) computers. The average speed-up when using 2 CPUs is over 90%.

2.3 Optimization of the voxel-coefficients

After the first beam profile optimization, the result is evaluated and the the voxel-dependent importance factors $\boldsymbol{\eta_v}$ (the voxel-coefficients) are adapted according to the violated constraints. A flow diagram of this algorithm is depicted in figure 1.

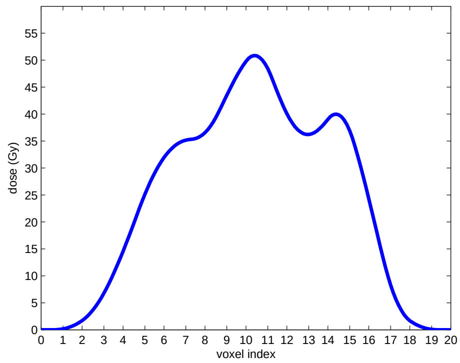
The constraints are divided into different constraint sets C_k, where k is a priority index. For example, the minimum PTV dose is usually put into C_1, because irradiating the PTV is the most important objective. The maximum dose in the PTV will usually be in C_2, and the organs at risk in C_3, C_4, ... (see table 1). In the second step of the optimization, the voxel-coefficients $\boldsymbol{\eta_v}$ are adapted only for the constraints in the first set C_k in which one or more constraints are violated.

volume	type	objective	critical	C_k
PTV	DV	100%	42.42 Gy	1
PTV	Max	47.78 Gy		2
Body	Max	47.78 Gy		2
Bowel	DV	25%	35 Gy	3
Bladder	DV	10%	40 Gy	3
Colon	DV	8%	40 Gy	3
Bowel	DV	37%	20 Gy	4
Bladder	DV	26%	20 Gy	4
Colon	DV	14%	20 Gy	4
Body	DV	16%	30 Gy	5

Table 1: Constraint list for the rectum case with a prescribed dose of 44.65 Gy. Five constraint sets are used. The smaller k, the higher is the importance of meeting that particular constraint.

To select the voxels of which the coefficient is adapted (i.e. increased), the dose of the particular volume is sorted in ascending order, see figure 2 [5]. For a maximum-dose constraint violation, a subset of the voxels exceeding the maximum-dose are adapted from high-dose to low-dose.

A dose-volume constraint requires a different approach. To lower a dose-volume value the voxels exceeding the critical dose are to be minimized. This can be achieved by increasing the voxel-coefficient of the voxels whose dose just exceeds the critical dose. This is depicted in the right part of figure 2 and visualised in figure 3.



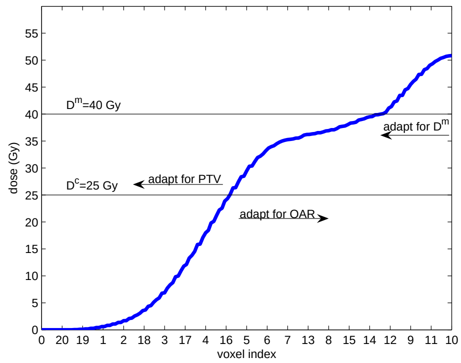


Figure 2: One dimensional example of the selection of voxels for adaptation of the (voxel-dependent) importance factor. The dose distribution (left) is sorted in ascending order (right). The first voxel exceeding the critical dose D^c is 16. For an OAR, the coefficient of voxels $16, 5, 6, \ldots$ are adapted. For a PTV voxels $4, 17, 3, \ldots$. If the coefficients are adapted because of a maximum-dose constraint violation, the voxels $10, 11, 9, \ldots$ are adapted.

One of the problems with this two-step approach is that the adaption of the voxels-coefficients does not have a direct relation to the criteria. It is not possible to predict the result of increasing the coefficient for a single voxel. For the maximum-dose voxel-adaption algorithm, experience learned that increasing the coefficient of the top 20% of the voxels violating the maximum-dose constraint by 0.3 proofs to be a good balance between decreasing the maximum-dose in the volume and over-emphasizing the voxels. For the dose-volume voxel-adaption algorithm, the coefficient of the first 20 voxels violating the constraint is increased by 1, unless there are less voxels violating this constraint. These settings result in a smooth convergence.

3 Results and discussion

The algorithm is demonstrated for a rectum cancer case. The prescribed dose is 44.65 Gy (19 fractrions of 2.35 Gy) and the patient is treated with four 18 MV beams and one 6 MV PA beam. 2297 beamlets of size 0.5×1.0 cm are used and 234 479 voxels of $0.4 \times 0.4 \times 0.5$ cm are considered in the patient volume. The volume-wide importance factors (see equation 1) for the PTV, bowel, bladder, colon and body are chosen 100, 10, 5, 5 and 1 respectively.

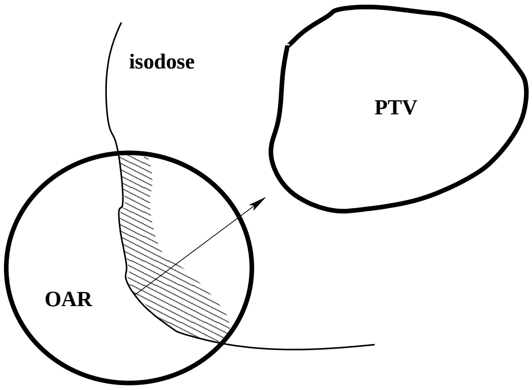


Figure 3: 2D example of voxel selection for voxel-coefficient adaption. The isodose line belongs to the critical dose D^c for the OAR. To reduce the amount of voxels exceeding the critical dose, the value of the voxel-coefficients of the voxels in the shaded area is increased. The result is that the isodose line moves closer to the PTV.

The constraints used are presented in table 1. Irradiating the tumour is the most important objective, so the constraint for the PTV is placed in the first constraint set (100% of the PTV receives a dose of 95% of the prescribed dose). The maximum PTV dose is 107% of the prescribed dose and is placed in the second constraint set, together with the maximum allowable dose in the body tissue (excluding PTV and OAR's). After the constraints in the first two sets are met, the algorithm tries to meet the constraints in the third set. This set contains the high dose constraints, since high dose in a volume has a larger impact on biological damage than a lower dose. The fourth constraint set contains constraints for the lower dose range. The reason that two DVH constraints are used for the OAR's is because the volumes are large. It turned out to be not efficient to only lower the high dose constraints: a lot could be gained in the low dose range as well. The last constraint on the body is to enforce more conformity and removes unnecessary dose in the unspecified tissue.

The optimization required 41 minutes for 273 iterations on a 2.4 GHz Intel Core2 processor. The results are presented in figure 4. The dose is highly conformal and 100% of the PTV is irradiated within the 95% – 107% dose constraint.

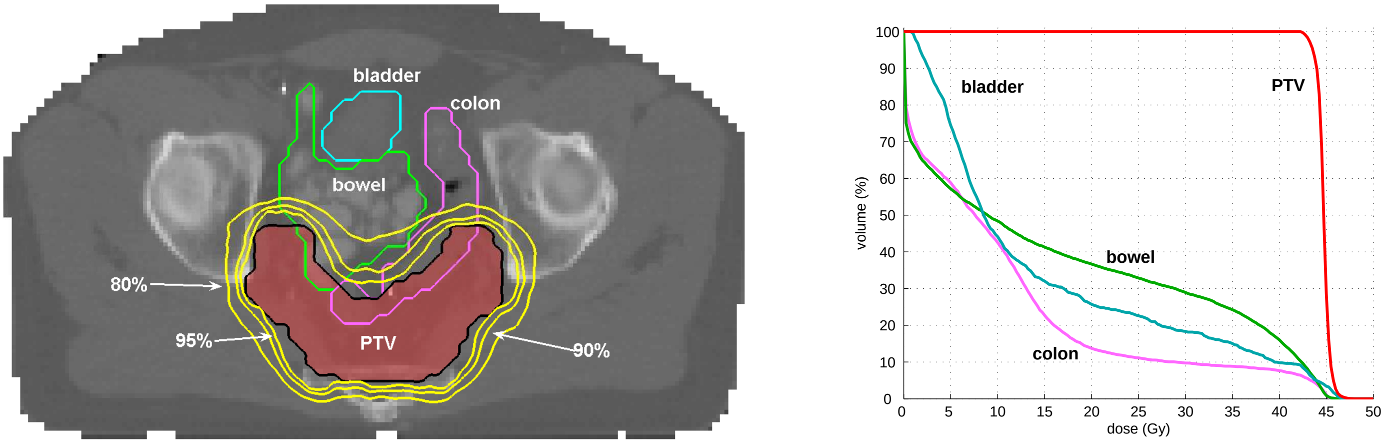


Figure 4: Left figure shows location of the volumes and the 80%, 90% and 95% isodose lines. The beams are placed at 85, 155, 180, 205 and 275 degrees. Right figure shows the dose-volume historgram.

One of the difficulties in inverse treatment planning is the choice of the constraints and the corresponding importance factors. If possible, an algorithm will provide a solution which satisfies the constraints, but not necessarily better. A lot of human interaction would be required to find the *most optimal* constraints. This is a result of the multi-criteria origin of the problem. In an upcoming paper we will present an algorithm that automatically finds the most optimal (i.e. Pareto optimal) constraints for an inverse planning process.

The time required for this optimization could be further reduced by using a parallel implementation, as mentioned in section 2.2.

4 Conclusion

In this paper we have presented an algorithm that effectively adapts voxel-dependent importance factors, used in combination with a quadratic objective function. This implemented two-step approach is capable of achieving highly conformal dose distributions.

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